NASA TECHNICAL TRANSLATION NASA TT F--11,717

EXPERIMENTAL FLOW FIELD INVESTIGATIONS NEAR THE SHARP LEADING EDGE OF A COOLED FLAT PLATE IN A HYPERVELOCITY, LOW DENSITY FLOW

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Translation of "Experimentelle Stromungsfelduntersuchungen nahe der scharfen Vorderkante einer gekuhlten,ebenen Platte in einer hypersonischen Stromung niedriger Dichte"
In: WGLR Meeting Report of the Technical Committee on Aerodynamics - Hypersound Aerodynamics, Deutsche Versuchsanstalt fur Luft- und Raumfahrt, Porz (West Germany), July 1967, pp 125-156

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. AUGUST 1968

EXPERIMENTAL FLOW FIELD INVESTIGATIONS NEAR THE SHARP LEADING EDGE OF A COOLED FLAT PLATE IN A HYPERVELOCITY, LOW DENSITY FLOW

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Abstract

Based on the findings of Report AEDC-TR-66-111, the flow field near the sharp leading edge of the cooled flat plate as manifested in different zones of hypersonic low-density flow is discussed. The parameter of viscous interaction

$$\bar{v}_{\infty,x} = M_{\infty} (c_{\infty}/Re_{\infty,x})^{1/2}$$

is of particular importance. A classification into zones is derived from the most recent available research.

List of Symbols

C_p	= pressure coefficient $p_W - p/q_{\infty}$
c_{∞}	= Chapman-Rubesin factor mu_W x T_{∞}/mu_{∞} T_W
D	= diameter
g	= gravitational acceleration
M	= Mach number
n	= exponent
P_r	= Prandt number
p_o	= constant from Ref. 1
p	= pressure
q_{∞}	= dynamic pressure in free jet 1/2 rho_{∞} u^2_{∞}
R	= gas constant
Re_{∞}	= unit Reynolds number rho_{∞} u_{∞}/mu_{∞}
$Re_{\infty,x}$	= Reynolds number rho_{∞} u_{∞} x/mu_{∞}
S	= Shock-form parameter
T	= temperature
T^*	= mean temperature from Ref. 39
u	= velocity
$\bar{v}_{\infty}$	= parameter of viscous interaction M_{∞} $(c_{\infty}/Re_{\infty})^{1/2}$
x	= distance from tip
y	= height above plate
alpha α	= thermal accommodation coefficient
DELTA Δ	= impact-layer thickness
delta δ	= boundary-layer thickness
delta* δ^*	= boundary-layer displacement thickness

delta$_o$	δ_o	= constant from Ref. 1
epsilon	ε	= density ratio
theta	θ	= angle of impact
kappa	$\varkappa$	= ratio of specific heat c_p/c_v
lambda	λ	= mean free path
mu	μ	= dynamic viscosity
rho	ρ	= density
sigma	σ	= energy accommodation coefficient
$\overline{\text{chi}}_\infty$	$\bar{\chi}_\infty$	= parameter of hypersonic interaction $M_\infty^3\ (c_\infty/Re_\infty)^{1/2}$

Indices

0	= at rest and/or Pitot
1	= ahead of impact
2	= beyond impact
A	= outer limit of impact
A'	= outer limit of impact from definition of maximum inclination
B	= inner limit of impact
ms	= maximum slope = maximum inclination
r	= general reference magnitude
s	= impact
x	= referenced to x
w	= wall
∞	= free jet

1. Introduction

The theoretical and experimental treatment of the flow field on a flat plate is one of the basic tasks of aerodynamics. Because of the relative simplicity of this body, it appears logical to begin with the investigation on the flat plate in research of a new zone of flow. Because of the practical importance of a cold surface, attention is directed to the states of a cooled wall with T_w/T_o very much less than 1.

2. Flow Regimes

Fig. 1 represents the model of plate flow at hypersonic velocity and low density. The deviations from conditions in the pure continuum concern the form of boundary layer, the effect of viscosity and the structure of the impact. The production of high enthalpies produces additional physical phenomena which are designated as vibration, dissociation, ionization, relaxation and recombination. Since only measurements from continuous nitrogen-operated hypersonic and low-density blast tunnels at T_o less than 3,000^o K are available, the following considerations will be restricted to the laminar hypersonic plate flow of low density without the action of real-gas effects. The designation of the flow zones from the trailing to the leading edge of the plate is derived from the most recent literature: "Zone of Interaction" (weak and strong interaction); "Mixed Zone" (incipient and fully merged layer or viscous layer); "Transition Zone" (transition flow); and "Disturbed Molecular Flow" (near free molecular flow).

The state of the friction-less outer layer between impact and boundary layer is determined, in the pure continuum, in accordance with the concept of Prandtl by the kind and geometry of the boundary layer formed in direct vicinity of the body. The distance x of a locus from the leading edge is introduced as characteristic measure in the Reynolds number. In Fig. 1, a position far downstream with high Reynolds number (large distance x) corresponds to the state of the pure continuum. Upstream from here in the direction of low Reynolds numbers, interactions between boundary layer and outer layer occur as a deviation from the boundary-layer theory. The form of the boundary layer induces the so-called hypersonic impact layer. On the other hand, the curvature of the impact causes a vortex which reacts on the boundary layer. A synopsis of the basic theoretical and experimental work as well as a detailed discussion of these interaction phenomena will be found in Ref. 1.

Upstream and in the direction of low Reynolds numbers follows the "mixed zone" in which outer layer and boundary layer intermix. It is also frequently designated as slip flow by which is meant that the classic condition of adhesion to the wall $\left|\left(\frac{\partial u}{\partial y}\right)_w = 0\right.$ is no longer satisfied. The additional utilization of a slip concept is intended to further expand the laws valid in the continuum. Actually, the conditions cannot be described by the introduction of a finite velocity and of a temperature jump on the wall. As indicated by the designation "mixed zone" selected for this zone, the primary effect is the intermixture of 2 layers which includes the slip phenomenon on the wall.

With further decreasing Reynolds number, a transition to "free molecular flow" becomes notable which can be described with the aid of the kinetic gas theory. The conditions in this zone of transition and also in the "mixed zone" have been investigated only in recent years. The present study reports on some special problems on the flat plate. A general review on the latest findings of low-density flow research in the last 2 zones inclusive of an extensive bibliography was given by Ref. 2.

With very low Reynolds numbers very near the leading edge, the distance x enters the order of magnitude of the mean free path lambda. This region is called "disturbed molecular flow". Ref. 3 analyzed in 1961 a zone of first and second impacts on the leading edge of a flat plate in low-density flow with the aid of the kinetic gas theory. In addition, reference is made to the work in Ref. 4 and 5.

"Free molecular flow" cannot be attained in practice even directly at the leading edge if the length of the plate is equal to or greater than one mean free path (Ref. 4, 5, 6). This is explained by the assumption that disturbances through reflected molecules from downstream (where disturbed molecular flow definitely must exist on the basis of the ratio lambda/x) may be transmitted at any time as far as x = 0. The conditions in "free molecular flow" as indicated in Ref. 1, 6, and 7 can be utilized as limit values.

3. Flow Model in Mixed and Transition Regime

At low Reynolds numbers, a phenomenon occurs on the wall which is designated as slip velocity and temperature jump. Whereas the effect of viscous forces and the heat transfer is considered as restricted in the continuum only to the boundary layer and the outer layer between boundary layer and impact is regarded as friction-less and uniform, an influence between boundary layer and outer layer occurs in the zone of interaction. This influence assumes such importance in the "mixed zone" that the 2 layers intermix. Accordingly, this zone is called "mixed zone". With decreasing Reynolds number, a velocity and temperature profile is less and less able to form and there is produced on the surface a finite velocity and a difference between gas temperature on the wall and wall temperature.

Ref. 7 considers the particles in a zone near the wall to have the extent of the mean free path and then classifies them as those which move in the direction of the wall with a mean tangential velocity u_w + lambda $(u/y)_w$ and those which move in the direction away from the wall with mean tangential velocity (1 - sigma) $[u_w$ + lambda $(u/y)_w]$ + sigma x 0. In the term of the particles coming from the wall, sigma designates the component of the completely diffuse-reflected and 1 - sigma for the component of the ideal reflected particles. The tangential velocity of the diffuse-reflected particles is zero in the mean. With these expressions, the slip velocity on the wall becomes:

$$u_w = \frac{2-\sigma}{\sigma} \lambda \left(\frac{\partial u}{\partial y}\right)_w$$

In the continuum, the slip velocity is negligibly small because of lambda different from 0. Toward free molecular flow, finite velocities are measured. It should also be noted that the existence of a temperature gradient additionally induces a velocity component of the order 3/4 mu/rho^T $(\partial T/\partial X)_w$ in the direction of the temperature rise. This is designated as thermal creep, a motion which occurs, in spite of equal pressure, only through a temperature difference (heat transfer). In the same way as the slip velocity on the wall is defined, a temperature jump can be described according to Ref. 7 as

$$T_w = \frac{2-\alpha}{\alpha} \; \frac{2\varkappa}{(\varkappa+1)P_r} \; \lambda \left(\frac{\partial T}{\partial y}\right)_w$$

in which alpha = extent of thermal accomodation of the gas to the state of the wall.

The slip concept discussed was intended to describe the observed deviations from the continuum with methods applicable to the continuum. This was based on the estimates in Ref. 1 of the order of magnitude in the Navier-Stokes equations. It represents a relatively thin impact in comparison to the distance of the impact from the wall. Ref. 9 arrives at a solution which is comparable to the one for wedge flow. In a publication of AIAA of 1964, Ref. 10 demonstrated that the introduction of the slip concept and solutions of first approximation cannot explain the actual measured deviations. The author doubts that the direct interaction of boundary layer and impact can in fact be represented satisfactorily by the Navier-Stokes equations.

Like Ref. 9, Ref. 6 was based on the same assumption but referred to a certain, although still negligible, impact thickness. In addition to the slip effect on the wall, it introduced a velocity and a temperature jump beyond the impact. The experiments do not confirm such a discontinuity. It was intended with this to take into account transport processes beyond the impact.

Another model resulted from the work of Ref. 11 and 12. These authors define a distance from the leading edge by which the formation of the impact is delayed as a consequence of the slip. This produces an impact form with concave curvature at the leading edge of the plate (reverse curvature). Ref. 13, 14, 15 fall in the same category.

The first experimental work extending into the "mixed regime" was published in Ref. 11 to 23. These reports relate primarily to the investigation of surface pressure and heat transfer. Some data on the entire flow field were available at the start of the "mixed regime" for insulated wall conditions (adiabatic wall) from Ref. 17, for the cooled wall from Ref. 20, and qualitatively as schlieren pictures from Ref. 17. A complete analysis of the flow field and of the surface pressure from the zone of interaction throughout the entire "mixed zone" is contained in Ref. 24, 25, 26, and 27. Ref. 28 and 29 simultaneously investigated, at a somewhat lower state of density, the "mixed zone" and the "transition zone". From the investigations reported in Ref. 25 and 28, there results, for the case of the cooled plate, a concept which in part decisively differs from the previously quoted theoretical work. Ref. 30 and 31, although believed to be provisional conclusions, support this concept. Reference is also made to the theoretical work published in Ref. 32, 33, 34 and 35 whose authors were familiar with the new flow model of Princeton and/or AEDC and took this into consideration.

4. Coefficients

For the zone of interaction, there results from Ref. 36, 37 and 38 very clearly a correlation of the measured findings through the parameter of hypersonic interaction

$$\bar{\chi}_{\infty,x} = M_\infty^3 \, (c_\infty / Re_{\infty,x})^{1/2} .$$

in which M_∞ = Mach number of free jet; $Re_{\infty,x}$ = the Reynolds number of the free jet referenced to the distance x; and

$$c_\infty = \frac{\mu_W}{\mu_\infty} \cdot \frac{T_\infty}{T_W}$$

= Chapman-Rubesin factor. Where local similarity exists, the Chapman-Rubesin factor can be improved to

$$c^* = \frac{\mu^*}{\mu_\infty} \cdot \frac{T_\infty}{T^*}$$

by the assumption of a mean temperature

$$T^* = T_o \left[1 + 3 \, T_W/T_o\right] /6$$

This method was proposed by Ref. 39. A synopsis of the zone of weak and strong interaction is contained in Ref. 1.

As was demonstrated for Mach numbers around 20 in Ref. 9 and later for Mach numbers around 10 in Ref. 28 (Fig. 18), $\overline{chi}_{\infty}/x$ loses its significance for values very much larger than 40 and/or 10, i.e. lie in the "mixed regime". There is valid here the parameter of viscous interaction

$$\bar{v}_{\infty,x} = M_\infty \, (c_\infty/Re_{\infty,x})^{1/2} .$$

which is frequently also called rarefaction parameter. Ref. 40 derived $v_{\infty,x}$ from the Knudsen number in 1963. From the kinetic gas theory and with the assumption of a completely elastic impact (billiard-ball model), the author formed

$$\lambda_r = \left(\frac{\pi \cdot x}{2}\right)^{1/2} \cdot \left(\frac{\mu_r}{\varrho_r \cdot a_r}\right)$$

in which r = general reference parameter, a Knudsen number

$$\frac{\lambda_r}{x} = \left(\frac{\pi x}{2}\right)^{1/2} \left(\frac{\mu_r}{\mu_\infty} \, \frac{\varrho_\infty}{\varrho_\infty} \, \frac{a_\infty}{a_r}\right) \quad \frac{M_\infty}{Re_{\infty,x}}$$

which can be rewritten as

$$\frac{\lambda_r}{x} = \left(\frac{\pi x}{2}\right)^{1/2} \quad \frac{p_\infty}{p_r} \left(\frac{T_r}{T_\infty}\right)^{3/2} \quad \frac{M_\infty c_\infty}{Re_{\infty,x}}$$

With the relations of viscous interaction from Ref. 1

$$\frac{\delta^*}{x} = \frac{\delta \bar{\chi}_{\infty,x}^{1/2}}{M_\infty}$$

$$\frac{p_\infty}{p_r} = \frac{1}{p_0 \bar{\chi}_{\infty,x}}$$

$$p_0 = \frac{9}{32} \varkappa(\varkappa+1)\delta_0^2$$

and with the aid of a simplified hypersonic impact relation (general parameter of reference r here signifies beyond the impact s)

$$\frac{T_s}{T_\infty} = \frac{\varkappa-1}{\varkappa+1} \quad \frac{p_a}{p_\infty} = \frac{\varkappa-1}{\varkappa+1} p_0 \bar{\chi}_{\infty,x}$$

the Knudsen number then becomes

$$\frac{\lambda_a}{\delta^*} = 0{,}664 \frac{\varkappa(\varkappa-1)^{3/2}}{\varkappa+1} \bar{v}_{\infty,x}^2$$

With a similar relation, Ref. 40 also establishes a correlation between the impact-layer thickness and $\bar{v}_{\infty,x}$. Since $\bar{v}_{\infty,x}$ takes into account both the slip effect as well as the increase of impact-layer thickness, the utilization of this parameter in the "mixed zone" is appropriate.

From the kinetic gas theory, it is possible to formulate, together with the assumption of a completely diffuse reflection of the particles on the wall, a combination of coefficients which is valid in "free molecular flow".

In the Knudsen number

$$\frac{\lambda r}{x} = \left(\frac{\pi\varkappa}{2}\right)^{1/2} \frac{p_\infty}{p_r} \left(\frac{T_r}{T_\infty}\right)^{3/2} \frac{M_\infty c_\infty}{Re_{\infty,x}}$$

we can introduce from Ref. 1 and 6 under such conditions and for r (here = wall):

$$\frac{p_w}{p_\infty} = \frac{1}{2}\left[1 + \left(\frac{T_w}{T_\infty}\right)^{1/2}\right].$$

This furnishes a Knudsen number

$$\frac{\lambda_w}{x} = (2\pi\varkappa)^{1/2} \frac{M_\infty c_\infty / Re_{\infty, x}}{(T_\infty/T_w)^{3/2} + (T_\infty/T_w)}$$

Essentially, there consequently exist a dependence on $M_\infty/Re_{\infty,x}$ and (T_∞/T_w). In order to come as close as possible to "free molecular flow" in "disturbed molecular flow", we need the combination of high Mach number and low Reynolds number. The combination of coefficients here indicated should be regarded only as tendency because completely diffuse reflection obviously cannot be attained in "disturbed molecular flow". Moreover, considerable caution must be exercised in the use of Knudsen numbers in this zone as demonstrated in Ref. 41 and 42.

5. Pitot-Pressure and Heat-Resistance Profiles

This chapter is intended to present an idea of the model from Ref. 28 and 29. The measurements were carried out in 1965 in the electric-arc heated vacuum wind tunnel of AEDC. The tunnel was described in Ref. 41, 42 and further in Ref. 28. In accordance with the classification of zones in Fig. 1, Fig. 2 presents as example the change of the Pitot-pressure profile from the pure continuum on the right to the mixed and transition zone on the left.

The deviations from the classic boundary-layer theory are caused by the interaction between boundary layer and outer layer. The character of impact (thin impact, force from Rankine-Hugoniot) remains preserved. However, negligible effects (finite impact thickness, slight slipping on the wall) already exist here.

The next characteristic change results in the "mixed zone" through the interaction of impact and/or impact layer with the zones ahead of and beyond the impact. The impact itself increases in thickness and decreases in intensity. Instead of the terms impact layer, outer layer and boundary layer, we can only speak of a profile width characteristic points.

In the direction of "disturbed molecular flow", this profile flattens out increasingly and finally assumes the value of the free jet. The slip along the wall appears to be only the consequence of the intensified mixing process so that the introduction of a slip concept in the laws of continuum no longer suffices for the actual conditions. The concepts of the customary compression impact are no longer valid for this case. Of particular importance appears to be the impact thickness $DELTA_{ms}$ (ms = maximum slope = maximum inclination) which is defined in Fig. 2.

Let us now turn to the measurements in the flow field in order to impart a quantitative idea. In Fig. 3 (Fig. 9 from Ref. 28), 4 Pitot-pressure profiles are plotted on the left and 4 hot-wire resistance profiles on the left. The Pitot pressures were corrected in regard to the effects of temperature, viscosity, geometry of incidence and heat transfer falsifying the measurements. This was based on the work in Ref. 43, 44 and 45. The free-jet state and the assignment of the profiles to $\overline{v}_{\infty,x}$ and/or $\overline{chi}_{\infty,x}$ are shown at the top of Fig. 3. The respectively highest profile belongs to a state in the center of the "mixed zone" which corresponds to the rear part of the plate investigated. The flattest profile belongs to a state far in the "transition zone", almost in the "disturbed molecular flow".

The measured findings clearly show the qualitatively discussed effects of intermixing and weakening of the impact with increasing rarefaction. In the direction of the leading edge of the plate, the intensity of the impact rapidly decreases and becomes insignificant with the flattening of the profiles. This occurs when the distance x becomes comparable to the mean free path lambda$_{\infty}$ within the range of impacts of the first order. There are further noticeable on the surface the high pressures from which slip velocity and temperature jump can be determined from knowledge of the measured statistical surface conditions.

The profiles near the leading edge -- for which $\overline{v}_{\infty,x} = 1.507$ was selected in Fig. 3 -- show a more pronounced character in the hot-wire resistance than in the Pitot-pressure measurements. This is explained by the fact that the intake diameter of the Pitot-pressure probe (0.5 mm) is about 50 times greater than the diameter of the hot wires. Profile-flattening measurement falsifications produced at the pressure probe from the interaction between the impact layer ahead of the probe with the plate surface and the impact layer from the plate flow, obviously had a strong effect.

The hot-wire probe furnishes considerably more exact findings because it reacts, in the investigation of the various regimes referenced to the wire diameter, as in "free molecular flow" (Knudsen number lambda$_{\infty}$/D = 80.

6. Impact Layer

The characteristic points of the profiles designated in Fig. 3 as y_A, y_A' and y_B can be plotted in a true y - x picture. Fig. 10 in Ref. 28 compares the respective profile tip y_B with the theoretical work in Ref. 9, 6, 46 and 34. It is significant that the first of the 3 references was based on the customary assumptions of a thin impact. Ref. 34 and 35 utilized an integral method for the "mixed zone" and employed a linear increase of slip velocity. The inter-relation y different from x^n is clearly shown in Fig. 11 from Ref. 28. It is of interest that the trace of the center of impact y_{ms} different from $x^{0.73}$ agrees rather well with the prediction of Ref. 47 which is valid for a thin impact.

The geometry of the impact layer is determined by the thickness and by the position relative to the plate. If we express the thickness DELTA$_{ms}$ relatively to the distance y_{ms} (center of impact to plate) over $\overline{v}_{\infty,x}$, we then obtain the plotting given in Fig. 4 (Fig. 12 from Ref. 28). It would seem evident that the intermixing process increases with increasing $\bar{v}_{\infty,x}$. As maximum in the ideal case, $DELTA_{ms}/y_{ms} = 2$ can be attained when the impact layer $DELTA_{ms}$ -- determined with maximum inclination -- extends from the free jet to the plate surface. At $v_{\infty,x}$ between 0.15 and 0.2, the impact thickness begins to be significant as shown in Ref. 24 to 27.

The position of the center of impact above the plate surface divided by the distance x is shown to be a useful combination as a function of the parameter of the viscous interaction $\bar{v}_{\infty,x}$. As will be seen from Fig. 5, the measured findings from Ref. 28 and 29 agree very well with the measurements of Ref. 24, 25, 26, 27, 30 and the lowest branch of Ref. 31. The upper branch of Ref. 31 is due to the very uncertain evaluation of almost flat profiles and should therefore cause no particular surprise. The dotted curve of Ref. 17 can probably be explained through the insulated wall conditions (adiabatic wall) of this investigation. The reaction of y_{ms}/x to low $\bar{v}_{\infty,x}$ within the zone of interaction (lower left edge) is discussed in Fig. 14 of Ref. 25.

If we relate the relative thickness datum $DELTA_{ms}/y_{ms}$ to the relative position datum y_{ms}/x, the representation in Fig. 6 then results. This ratio shall be designated as shock form parameter because it contains all the factors determining the geometric form. The shock-form parameter S should just rise from zero in the zone of viscous interaction and should again drop back to zero toward the "free molecular flow" under elastic collision of the particles. $DELTA_{ms}/y_{ms}$ tends toward 2 in the limit case of high $\bar{v}_{\infty,x}$ values; y_{ms} is assumed as finite in the order of magnitude of $lambda_{\infty}/2$ in accordance with the discussed assumptions of a never perfect "free molecular flow" even at very small distances x. In practice, $\bar{v}_{\infty,x}$ values different from zero will be manifested for S at a larger parameter. In the "mixed zone", the shock-form parameter shows a rise which is produced by the formation of the thick impact layer. The "zone of transition" and the "disturbed molecular flow" are characterized by a slight decrease resulting from the gradual accommodation of the flow character to the "free molecular flow". The measured findings from Ref. 25, 28, 29 and 30 acquire a logical order. The decrease of the shock-form parameter S in Ref. 31 for $\bar{v}_{\infty,x}$ greater than 0.4 results from the behavior in the preceding picture. Moreover, all of the data in Fig. 6 are experimental data; presently published theoretical work would indicate S = o or S approximately 0.

The greater the intermixture of the impact with the layer ahead of and beyond the impact which produces the impact layer, the weaker it becomes. Fig. 7 (Fig. 4 from Ref. 28) demonstrates the Pitot-pressure difference due to the impact layer. Referenced to the experimentally determined impact angle $beta_B$ at the end of the impact, the Rankine-Hugoniot value has been plotted but it is apparent that the measurements lie already far from the latter. The actual intermixing appears to take place in the range for $\bar{v}_{\infty,x}$ less than 0.5 if we the deviation of the ratio $p_{2,0}/p_{\infty,0}$

from the Rankine-Hugoniot value is made the indicator of this effect. A mixing process with very much lesser decomposition of the pressure ratio follows for $\bar{v}_{\infty,x}$ greater than 0.5.

7. Slip Velocity

The state on the surface characterized by the slip velocity which is indicated both by extrapolation of the velocity profiles calculated from the measured data and by direct extrapolation from the Pitot-pressure profiles in Fig. 8 (Fig. 13-b from Ref. 28). In the values determined from the velocity profiles, there are involved purely qualitative data because, in the calculation of the velocity profiles, a rectilinear impact ahead of the Pitot probe was assumed for lack of precise data. Consequently, these slip velocities should be less than those expected toward higher $\bar{v}_{\infty,x}$. The calculation of slip velocity from direct extrapolation of the Pitot pressures on the wall, from the surface pressure and from the surface temperature of the plate was derived in Ref. 28 as follows in which p_w = statistical pressure of the gas on the surface and $p_{2,ow}$ - rho $2w^u 2w^2/2$ = statistical pressure of the gas at the surface beyond the impact: impulse theorem:

$$p_{2,ow} - \frac{\rho_{2w}\, u_{2w}^2}{2} + \rho_{2w}\, u_{2w}^2 = p_w + \rho_w\, u_w^2$$

continuity:

$$\rho_{2w}\, u_{2w} = \rho_w\, u_w$$

combination:

$$\frac{p_{2,ow}}{\rho_w\, u_w^2} + \frac{\epsilon_w}{2} = \frac{p_w}{\rho_w\, u_w^2} + 1$$

slip equation:

$$u_w = \left[\frac{p_{2,ow} - p_w}{\rho_w\left(1 - \frac{\epsilon_w}{2}\right)}\right]^{1/2}$$

with

$$\epsilon_w = \frac{\rho_w}{\rho_{2w}} = \frac{u_{2w}}{u_w} = \frac{(\varkappa - 1) M_w^2 + 2}{(\varkappa + 1)\, M_w^2} \qquad \text{(rectilinear-impact relation)}$$

$$\rho_w = \frac{p_w}{g\, R\, T_w} \qquad \text{(equation of state)}$$

$$M_w = f\,(p_{2,ow}\,;p_w) \quad \text{(Rayleigh-Pitot formula).}$$

The objection here is that a temperature jump was not taken into account and that the calculation of the density ratio, epsilon_w is based on the assumption of a strong rectilinear impact. The influence on measured data in Fig. 8 can be considerable in regard to the temperature jump but not in regard to the epsilon_w determination. Accordingly, this method is regarded as more accurate than the extrapolation from the velocity profiles. The available solutions from Ref. 9 and 35 have also been plotted in Fig. 8.

8. Surface Pressure

The correlation of the surface-pressure data has been shown from the work in Ref. 19, 28 and 29 as appropriate in the form pressure coefficient C_p as a function of $\bar{v}_{\infty,x}$ in which

$$C_p = \frac{p_w - p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2} = \frac{2}{\varkappa M_\infty^2}\left(\frac{p_w}{p_\infty} - 1\right)$$

In the zone of interaction, the plotting shows the following dependence:

$$C_p = \frac{2}{\varkappa}\left(\text{const}\ \bar{v}_{\infty,x} - \frac{1}{M_\infty^2}\right)$$

because of

$$\frac{p_w}{p_\infty} = \text{const}\ \bar{\chi}_{\infty,x}\ .$$

In the transition zone, there is formed a "local plateau", so designated in Ref. 23 which investigated the insulated wall conditions. In the "free molecular flow" where we can postulate

$$\frac{p_w}{p_\infty} = \frac{1}{2}\left[1 + \left(\frac{T_w}{T_\infty}\right)^{1/2}\right]$$

the pressure coefficient becomes

$$C_p = \frac{2}{\varkappa M_\infty^2}\left\{\frac{1}{2}\left[1 + \left(\frac{T_w}{T_\infty}\right)^{1/2}\right] - 1\right\}$$

In Fig. 9 (Fig. 19 from Ref. 28), the available data on the surface pressure of cooled walls are combined. The black measured points were derived from Ref. 28 and 29 and the other data from Ref. 12 and 18 (in the corrected form of Ref. 32, 19 and 22).

9. Conclusions

The preceding chapters discussed in detail the essential properties of the flow field around a flat plate in rarefied gases. It was subdivided in considerations on the impact geometry (thickness and position), force of impact and surface conditions (slip velocity and surface pressure). If all detail informations are summarized and plotted over the parameter of the viscous interaction $\bar{v}_{\infty,x}$ as in Fig. 10, a subdivision into zones from the jointly derivable gradients should be possible. The hachured ranges for $\bar{v}_{\infty,x}$ = 0.15 + 0.17 and 0.45 + 0.55 shall delimit the "zones of interaction", the "mixed zone" and the "transition zone". The difference between "transition regime" and "disturbed molecular flow" becomes apparent beyond $\bar{v}_{\infty,x}$ greater than 2.0 and still requires more accurate explanation.

In connection with the considerations in the Section on Coefficients, it should be noted that the plotting over $\bar{v}_{\infty,x}$ is not exact in the "transition zone" and in the "disturbed molecular flow". As shown by the tendency to "free molecular flow", a combination of $\bar{v}_{\infty,x}$ with T_W/T_∞ appears to be more appropriate. In conclusion it should be noted that, in the plasma wind tunnel of the Institute for Applied Gas Dynamics of DVL in Porz-Wahn, the studies on the flat plate are being continued.

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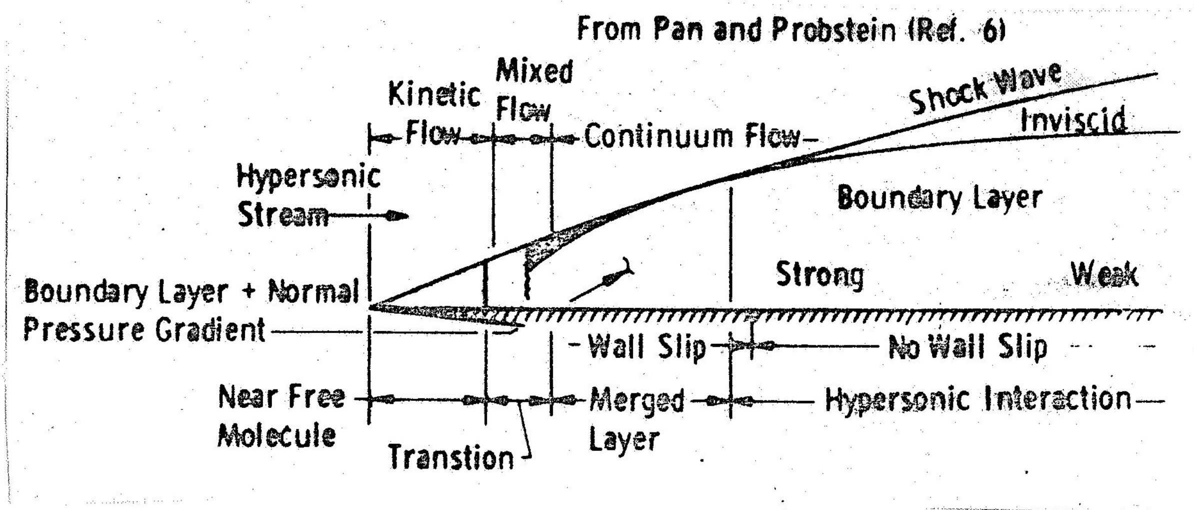

Figure 1. Hypersonic Rarefied Flow at the Sharp Edge of a Flat Plate

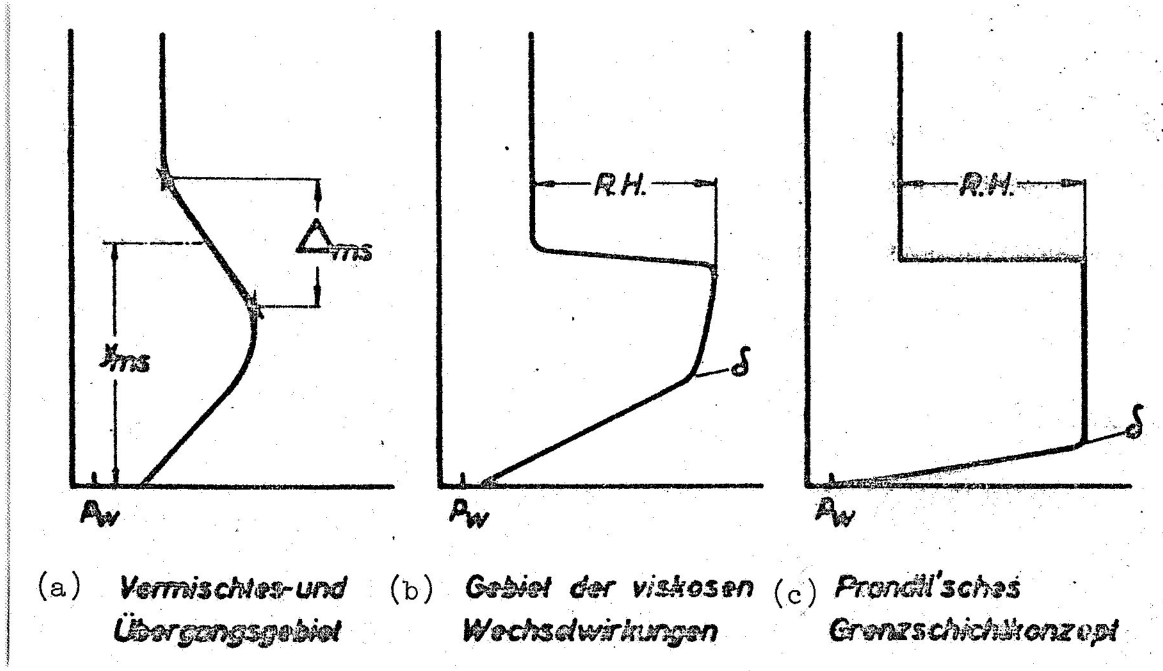

Figure 2. Pitot-Pressure Profiles in Different Flow Zones. a -- mixed and transition zone; b -- zone of viscous interaction; c -- boundary-layer concept of Prandtl.

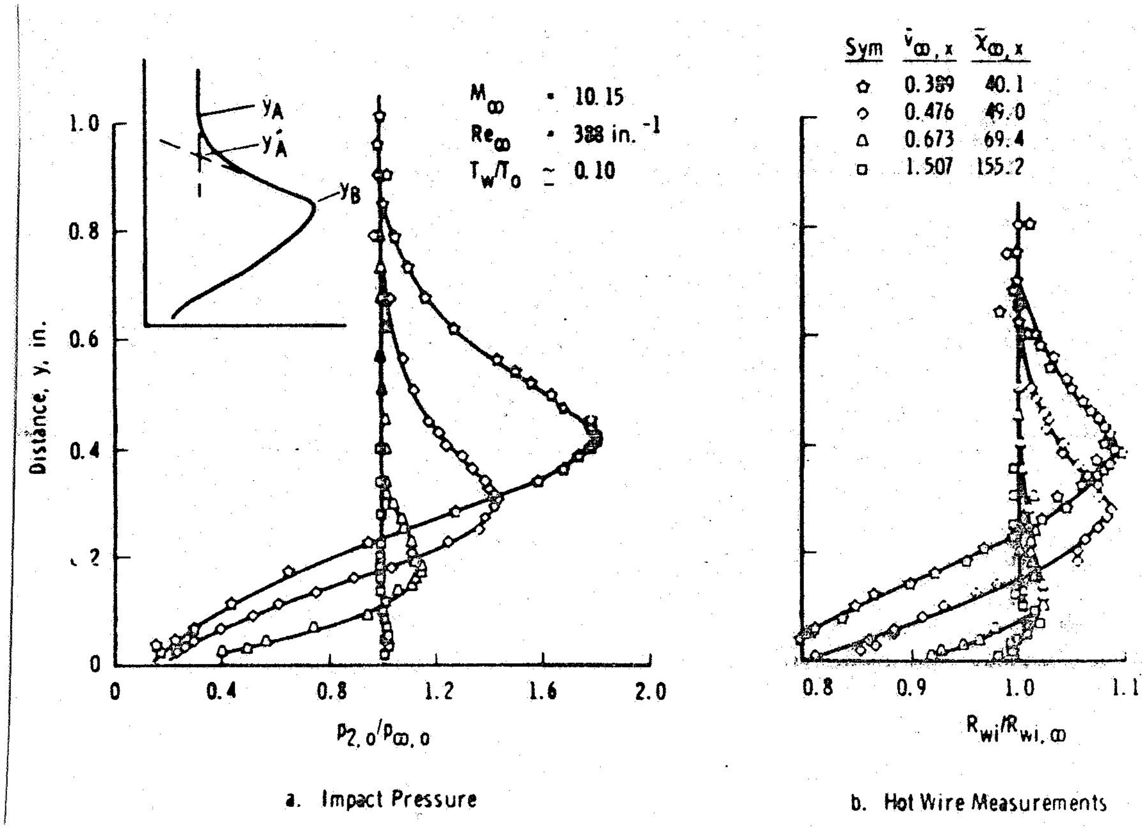

Figure 3. Flow-Field Investigations with Pitot-Pressure and Hot-Wire Resistance

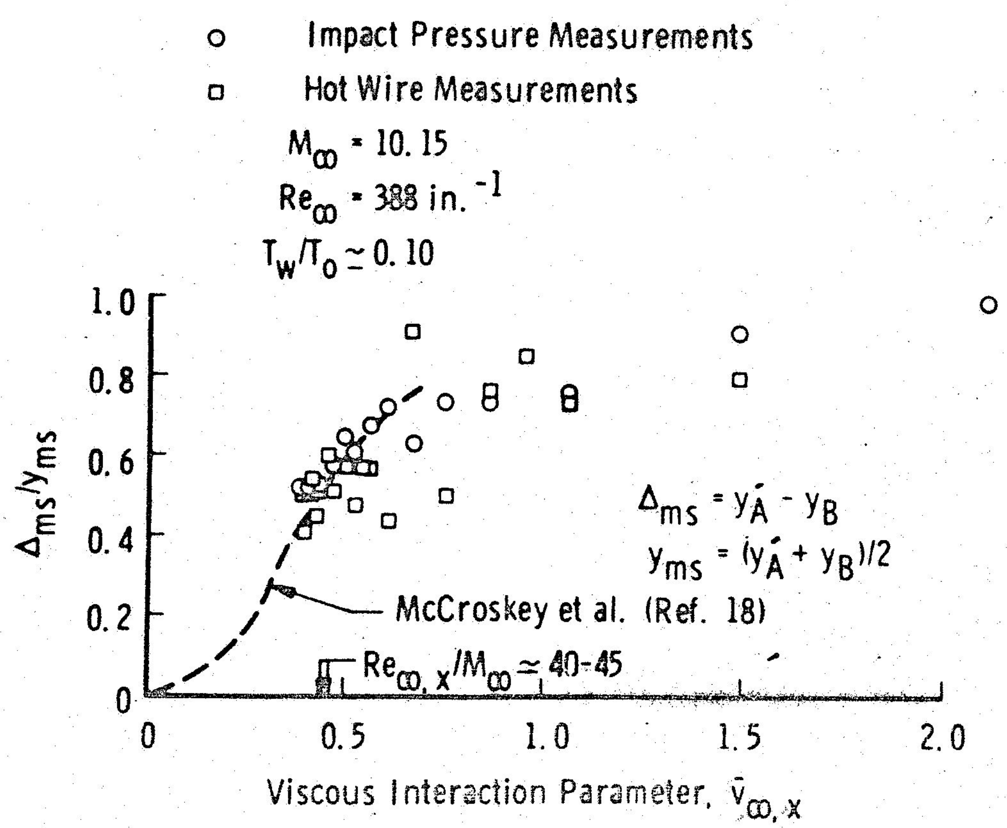

Figure 4. Impact Thickness as a Function of $\bar{v}_{\infty,x}$.

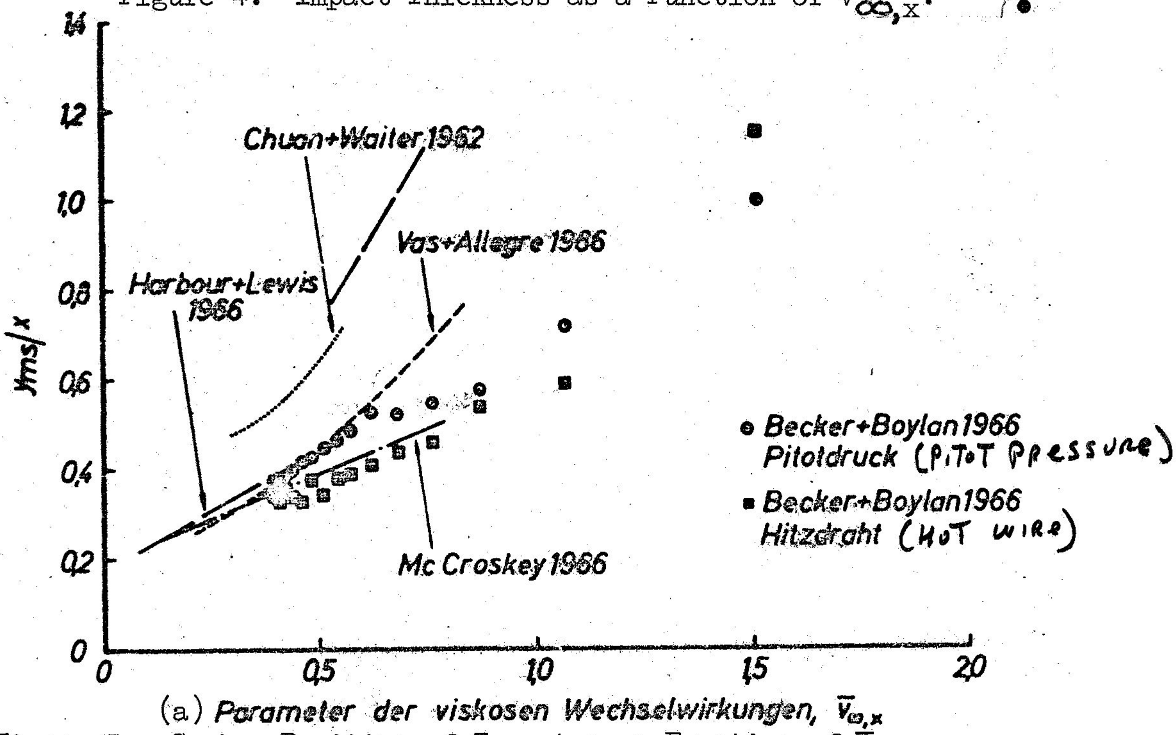

Figure 5. Center Position of Impact as a Function of $\bar{v}_{\infty,x}$. a = parameter of viscous interaction $\bar{v}_{\infty,x}$

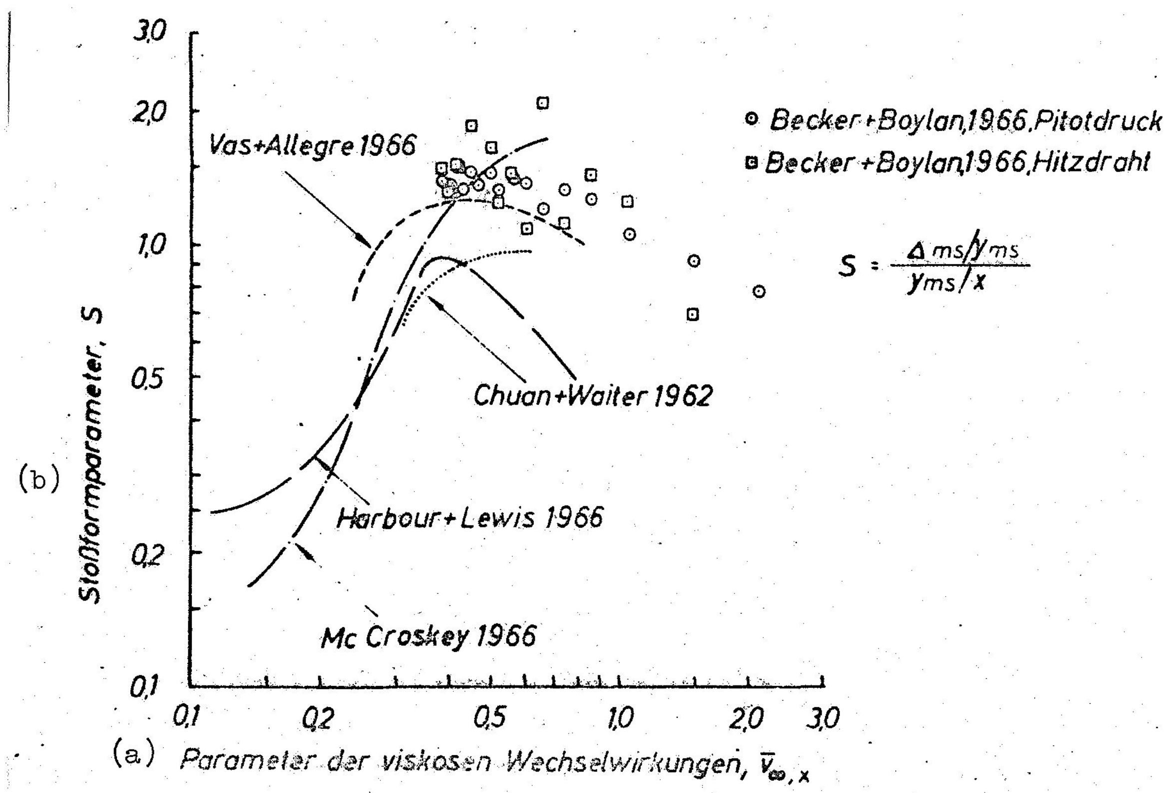

Figure 6. Impact-Form Parameter as a Function of $\bar{v}_{\infty,x}$.
a = parameter of viscous interaction $\bar{v}_{\infty,x}$;
b = shock-form parameter S.

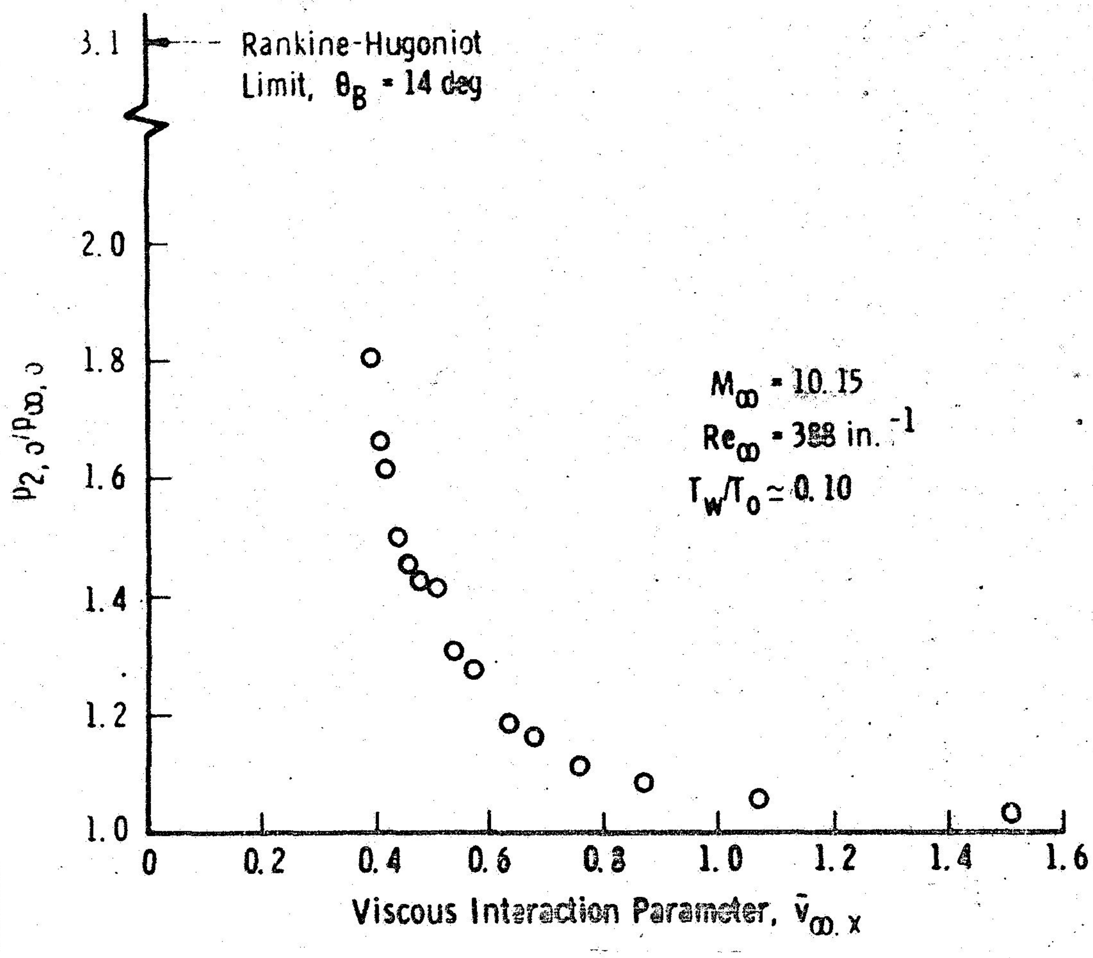

Figure 7. Impact Force as a Function of $\overline{v}_{\infty,x}$.

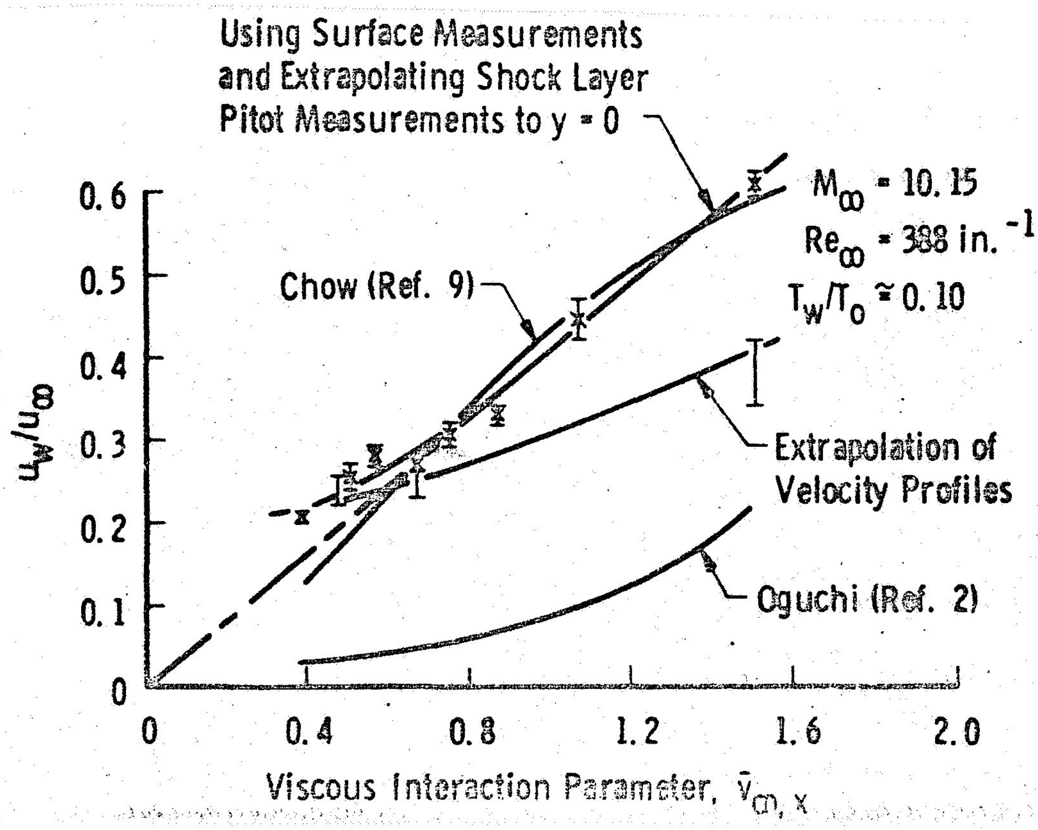

Figure 8. Caption Illegible.

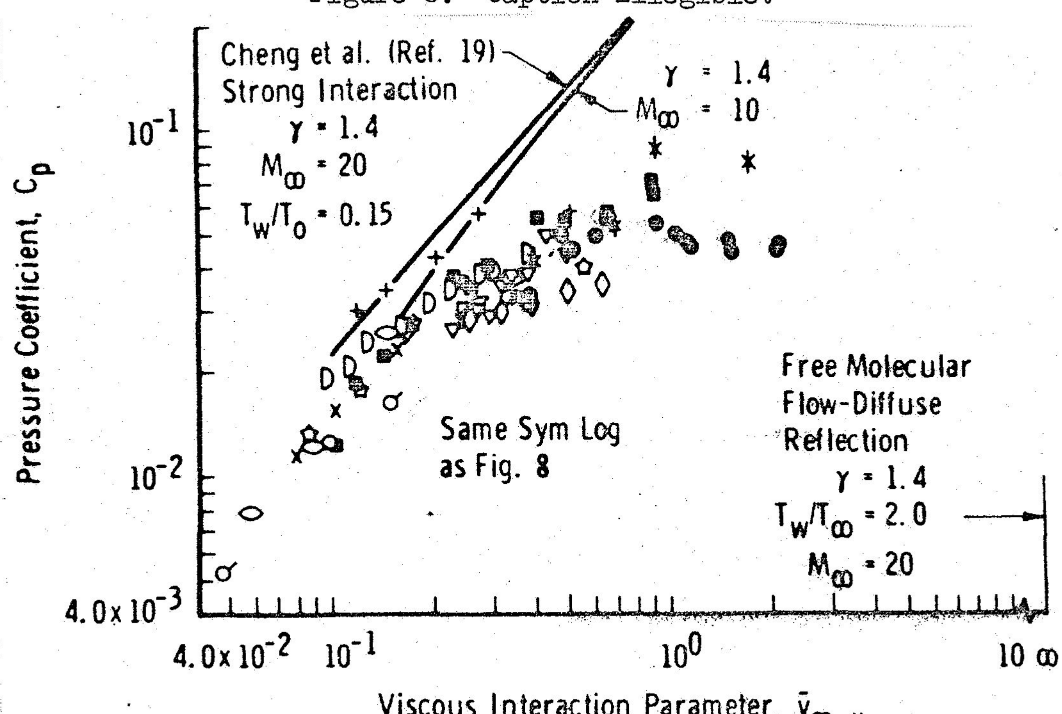

Figure 9. Pressure Coefficient C_p as a Function of $\bar{v}_{\infty,x}$.

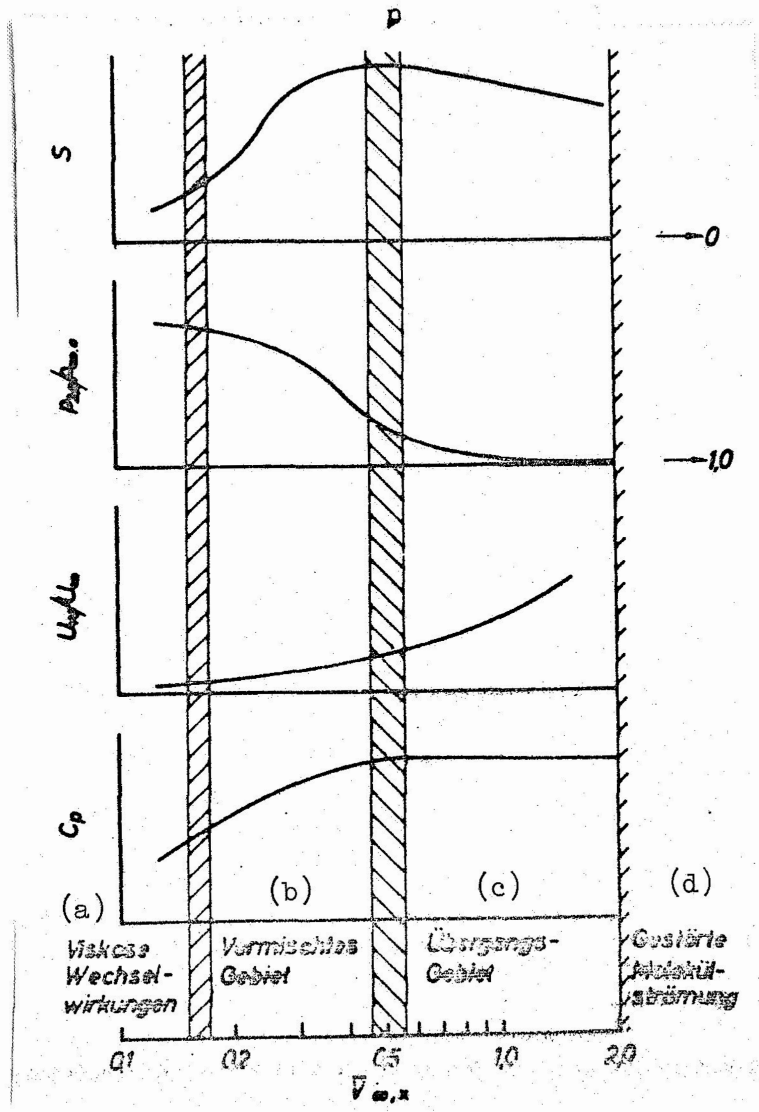

Figure 10. Tendencies in Different Flow Zones. a = viscous interaction; b = mixed zone; c = transition zone; d = disturbed molecular flow.

List of Participants in the Eleventh Meeting of the Committee on Aerodynamics on 9 December 1966

Subject: Hypersonic Aerodynamics

Amtsberg	Dipl.-Ing.	DFL
Arnold	Dipl.-Ing.	DFL
Becker	Dipl.-Ing.	DVL
Bichfeldt	Dipl.-Ing.	HFB
Cramer	Dipl.-Ing.	Bölkow
Deppe	Dipl.-Ing.	Bölkow
Diesinger	Dr.-Ing.	TH Aachen
Ebeling	Dipl.-Ing.	VFW
Eder	Dr.	EWR
Engelhard	Assessor	DFG
Eppler	Prof. Dr.-Ing.	Bölkow
Friedel	Dr.-Ing.	Dornier
Frohn	Dr.	TH Aachen
Gersten	Prof. Dr.-Ing.	DFL
Grabitz	Dr.	MPI
Harms	Dipl.-Ing.	AVA
Harms	Ing.	Dornier-System
Hoicker	Dipl.-Ing.	EWR
Hornung	cand. phys.	TH München
Hummel	Dipl.-Ing.	DFL
Igenbergs	Dipl.-Ing.	TH München
Kapp	Dipl.-Ing.	Dornier-System
Kausche	Dipl.-Ing.	DFL
Kretzschmar	Dipl.-Ing.	ONERA, Paris
Kun	Ing.	EWR
Laschka	Dr.-Ing.	VFW
Lessing	Dipl.-Ing.	Bölkow
Lipowski	Dipl.-Ing.	AVA
Liu	Dipl.-Ing.	Junkers
Löser	Dipl.-Ing.	ERNO
Lotter	Dipl.-Ing.	EWR
Lotz	Dr.-Ing.	Dornier
Lugt	Dr.-Ing.	Junkers
Mack	ORR	BMVtdg.
Mansouri	Dr.-Ing.	HFB
Mittelbach	Dr.	Bölkow
Oertel	Dr.	ISL
Paulig	Dipl.-Ing.	Junkers
Pfeiffer	Dr.-Ing.	DVL
Raldschmidts		EWR
Riegels	Dr.	AVA
Römer	Dipl.-Ing.	TH München

Rotta	Ing.	AVA
Ruden	Prof.	VFW
Schepers	Dipl.-Ing.	DVL
Schmidt	Dipl.-Phys.	Bölkow
Schlinsdegg	Dipl.-Ing.	Junkers
Schneider	Dr. techn.	DVL
Seifferth	Dipl.-Ing.	EWR
Söffker	Dipl.-Ing.	ERNO
Strassemeyer	Dipl.-Phys.	TH Karlsruhe
Strauber	Dipl.-Ing.	TH Darmstadt
Stursberg	Dipl.-Ing.	DVL
Timme	Dr.-Ing.	DVL
Truckenbrodt	Prof. Dr.-Ing.	TH München
Viehweger	Dipl.-Ing.	DVL
Wagner	Dipl.-Ing.	TH Darmstadt
Wilckens	Dipl.-Ing.	ERNO
Winkler	Dipl.-Ing.	TH München
Witte	Dipl.-Ing.	TH Darmstadt
Wolf	Dipl.-Ing.	VFW
Wuest	Dr.	AVA
Wulf	Dipl.-Ing.	AVA
Wyborny	Dipl.-Ing.	DVL
Zeller	Dr.	TH Aachen